SUPERCONVERGENCE AND ERROR ANALYSIS FOR THE *hp*-DISCONTINUOUS FINITE ELEMENT METHOD

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The discontinuous Galerkin method (DGM) is an appealing approach to address problems having discontinuities, such as those that arise in hyperbolic conservation laws. The DGM uses a discontinuous finite element basis which simplifies *hp* adaptivity and leads to a simple communication pattern across faces that makes it useful for parallel computation. In order for the DGM to be useful in an adaptive setting, techniques for estimating the discretization errors should be available both to guide adaptive enrichment and to provide a stopping criteria for the solution process. We will show that the $p-$degree DG finite element solution for hyperbolic problems exhibits superconvergence at the roots of Radau polynomials of degree $p+1$ with the fixed endpoints selected at the downwind boundary of each quadrilateral element. We also show that the DG solution has strong superconvergence on average at the outflow boundary. We discuss an extension of these results to locally discontinuous finite element solutions of convection-diffusion problems. We use our superconvergence results to construct asymptotically exact a posteriori error estimates for first-order hyperbolic and convection-diffusion problems. Finally, we present numerical results for several computational examples with both continuous and discontinuous solutions that validate our theory.